

DISCUSSIONS

'HUME'S THEOREM' CONCERNING MIRACLES

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In 'Hume's Theorem on Testimony Sufficient to Establish a Miracle' (*The Philosophical Quarterly*, 41 (1991), pp. 229–37), Jordan Howard Sobel proposes a Bayesian interpretation of Hume's famous 'general maxim' which ends Part I of his celebrated essay 'Of Miracles', Section X of the first *Enquiry*:

That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish; and even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior (*E* 115–16).¹

Sobel coins the name 'Hume's Theorem' for the first half of this general maxim (i.e., that part preceding the semicolon), and he interprets it as follows:

$$P(\alpha) > 0 \ \& \ P(A/\alpha) > \frac{1}{2} \rightarrow P(A) > P(\alpha \ \& \ \neg A)$$

where '*A* assert(s) what would be a miraculous occurrence, and . . . α assert(s) the occurrence of testimony for *A*' (p. 232).² Sobel provides a fairly technical eleven-line proof of this formula in an appendix, but although he welcomes the conclusion that Hume's maxim is a theorem of probability theory, he is clearly anxious to avoid the suggestion that it is merely trivial. Thus, for example, having recognized that his formula 'reduces to a near tautology . . . with cases of "known testimony" in which $P(\alpha) = 1$ ', he avoids this supposedly undesirable consequence by proposing a retrospective interpretation of the maxim, so that in such cases 'what Hume wanted to say . . . was something like this':

That no testimony that it is known for sure has been given is sufficient to establish a miracle, unless this testimony was, just before it became known for sure that it had been given . . . of such a kind that its falsehood was then more miraculous, than the fact which it endeavoured to establish.

¹ '*E*' signifies a page reference to the *Enquiry Concerning Human Understanding*, ed. L.A. Selby-Bigge and P.H. Nidditch (Oxford: Clarendon Press, 3rd edn, 1975).

² In elucidating his interpretation Sobel states that he has 'taken " ϕ unless ψ " in the sense of " $\neg \phi \vee \psi$ " or, equivalently, of " $\phi \rightarrow \psi$ "' (p. 232). But this is surely a slip, since it is clear that the last two formulae correspond not to ' ϕ unless ψ ' but to ' $\neg \phi$ unless ψ '.

Or in other words, 'Hume wanted the necessary condition for known testimony's being sufficient to establish a miracle, to be that it was, just before it became known, sufficient to do this' (p. 233).

One fundamental objection to Sobel's interpretation of Hume might be that it ignores the significance of Hume's words 'testimony . . . of such a kind . . .', which indicate that Hume is concerned with probabilities pertaining to *types* of testimony rather than tokens. These few words, however, are not decisive on this issue, and it may be that the choice between a 'type' and a 'token' interpretation can only be made by comparing together the best candidate of each sort. Despite my reservations I shall therefore focus here on more clear-cut objections to Sobel, and take for granted in most of what follows that a Bayesian 'token' interpretation such as he favours is indeed defensible.

My strategy for opposing Sobel's interpretation of Hume falls into two parts. I start by proposing an alternative formal interpretation of Hume's 'maxim' which, though in the same spirit as Sobel's, is considerably simpler, and I then explain why his own more elaborate interpretation seems by comparison unwarranted. Next I go on to deal with what, in the light of his remarks about avoiding a 'near tautology', would be the obvious objection from Sobel, that this simple interpretation is so crude that it could not possibly be what Hume had in mind, given the relatively sophisticated argument which precedes the statement of his maxim in Section X of the *Enquiry*. To parry this objection, I first show in passing that it cuts both ways, since Sobel's interpretation itself is a 'near tautology', and I end by explaining why I believe that Hume could after all have seen great philosophical and practical importance in a result which, when translated into the formal language of probability theory, is indeed quite trivial.

I

Using the same notation as Sobel's, my analogous but simpler formal interpretation of (the first half of) Hume's 'maxim' is as follows:

$$P(A/\alpha) > \frac{1}{2} \rightarrow P(A/\alpha) > P(\neg A/\alpha)$$

That is, the supposedly miraculous event A is probable on the basis of testimony α only if the probability of A given α is greater than the probability of $\neg A$ given α .³ If he was thinking probabilistically, Hume would certainly have assumed that α , being a 'matter of fact', must have an initial probability greater than zero (I shall accordingly assume that this is taken for granted in the rest of the discussion). And then the formula above is certainly trivial, since in this case it is obvious from the meaning of conditional probabilities that $P(A/\alpha)$ and $P(\neg A/\alpha)$ must add up to 1, a fact that is equally evident from their formal definition.⁴

³ Strictly ' A ' and ' α ' are not events but propositions, and so this sentence is imprecise. But where there is no risk of misunderstanding I shall continue for the sake of clarity to speak ambiguously of A as a purported miracle (or the proposition that A occurs), and α as the testimony for A (or the proposition that α is presented).

⁴ Sobel states the three standard axioms of propositional probability theory as follows

We can focus on the precise disagreement between this interpretation and Sobel's by formalizing Hume's words step by step and identifying the point at which the two interpretations begin to diverge. As a start, I assume that Sobel would be happy to paraphrase Hume as follows:

- (t) To establish some miracle A , testimony α (which testifies that A indeed occurred) must be such that its falsehood would be more miraculous than A itself.

This paraphrase may superficially appear obvious, so it is worth pointing out that it already takes for granted a 'token' interpretation of Hume's maxim. The alternative 'type' interpretation might be expressed as follows:

- (T) To establish some miracle A , testimony α (which testifies that A indeed occurred) must be of a kind k , where k is such that the falsehood of some item of testimony of that kind would be more miraculous than A itself.

For the reasons explained earlier, I shall not here attempt any further elucidation of (T) – a task which is surprisingly difficult except with respect to certain artificial examples (such as the one with which this paper ends). Here I shall instead concentrate attention on the relatively straightforward (t), which can easily be translated into formal probabilistic terms by interpreting 'establish' as 'render more probable than not', and 'more miraculous' as 'less probable'. These changes give us:

To render miracle A more probable than not, testimony α must be such that its falsehood would be less probable than A itself.

And semi-formally, this can be presented as follows:

- (H) $\text{probability}(A) > \frac{1}{2} \rightarrow \text{probability}(A) > \text{probability}(\alpha \text{ is false})$

I imagine that Sobel would be in agreement up to this point, and would be ready to concede that the ' $P(\alpha) > 0$ ' condition in his own formula is something which Hume would take for granted rather than anything which is stated explicitly in the text of the *Enquiry*. Omitting this condition, we have two structurally similar rival formal interpretations of the semi-formal Humean principle (H):

- (S) $P(A/\alpha) > \frac{1}{2} \rightarrow P(A) > P(\alpha \ \& \ \neg A)$

- (M) $P(A/\alpha) > \frac{1}{2} \rightarrow P(A/\alpha) > P(\neg A/\alpha)$

Both formulae take the antecedent of (H) to be concerned with a conditional probability: the probability that A occurred given that testimony α has been

(p. 232): '(i) probabilities are non-negative; (ii) the probability of any necessary proposition is 1; and (iii) the probability of a disjunction whose disjuncts are logically incompatible is equal to the sum of the probabilities of these disjuncts'. Axiom (iii) provides the crucial penultimate step in the following simple proof:

$$\begin{aligned} P(A/\alpha) + P(\neg A/\alpha) &= P(A \ \& \ \alpha)/P(\alpha) + P(\neg A \ \& \ \alpha)/P(\alpha) \\ &= [P(A \ \& \ \alpha) + P(\neg A \ \& \ \alpha)]/P(\alpha) = P(\alpha)/P(\alpha) = 1 \end{aligned}$$

presented. However whereas my own suggested formula (M) makes a similar assumption of conditionality about the probabilities that occur in the consequent of (H), Sobel's formula (S) interprets these as being non-conditional. This then, in essence, is the disagreement between us.

Unfortunately Sobel appears not to have considered the possibility of interpreting all three of the probabilities in Hume's maxim conditionally, for I can find nothing in what he says to support (S) as against (M). But there are at least two strong arguments on the other side, in favour of (M): one of these appeals to Hume's text alone, and the other also to logic.

The first argument against Sobel's interpretation is that in Part I of 'Of Miracles', which culminates with his maxim, Hume shows no interest whatever in the antecedent probability of the testimony α itself, but seems very clearly to take that testimony as 'given', being solely concerned to ask what conclusions can legitimately be drawn from it (thus, in particular, Sobel's convoluted retrospective adaptation of Hume's maxim for dealing with cases of 'known testimony' seems entirely *ad hoc*, with no basis whatever in the text itself). Hume does, of course, make frequent reference to the likelihood of the testimony's being false, but in context it is surely more natural to interpret this likelihood as $P(\neg A/\alpha)$ – the probability that α is false given that α has been presented – rather than $P(\alpha \ \& \ \neg A)$ – the probability that α should be presented and also be false. Moreover if Hume did indeed mean the latter, then his failure even to discuss the probability of the testimony's being presented would be quite inexplicable.

My second argument is anticipated by Sobel himself, though his response to it is somewhat perplexing. His problem is that although formula (S) is indeed a theorem of probability theory, the corresponding biconditional

$$(S') \quad P(A/\alpha) > \frac{1}{2} \leftrightarrow P(A) > P(\alpha \ \& \ \neg A)$$

is not, since it can fail to be true if, for example, α is antecedently even more improbable than A but is utterly worthless as testimony.

Consider, for instance, the entirely bogus but wealthy 'Psychic Sam', who in order to further his reputation adopts a policy of regularly taking out advertisements in a wide range of local newspapers, each of which purports to predict the result of a local weekly lottery (the idea being that Sam's many failures will be overlooked as long as the advertisements are suitably discreet, whereas a single success could be publicized to make his name). Suppose now that I am the last person to buy a ticket before the Little Puddleton lottery, and receive number 3247, although 9999 tickets were originally available. In this case it may well be more likely that I will win the lottery (1 in 3247) than it is that Sam will have predicted my success (say, 1 in 9999), but this clearly does nothing whatever to add credibility to his testimony.

Hence Sobel's formula specifies only a necessary and not a sufficient condition for testimony to establish a miracle. However in the very paragraph that contains his 'maxim', Hume's words indicate no fewer than three times that he himself took it to provide both necessary and sufficient conditions,⁵ a regrettable lapse on Sobel's

⁵ (a) 'even in that case . . . the superior . . . gives us an assurance'; (b) 'I weigh the one

interpretation, but perfectly understandable on mine, since formula (M) corresponds to the biconditional theorem

$$(M') \quad P(A/\alpha) > \frac{1}{2} \leftrightarrow P(A/\alpha) > P(\neg A/\alpha)$$

Sobel is aware of this problem, and he suggests what he takes to be an improvement on Hume with a biconditional formal maxim named 'Testimony for Miracles' (p. 234), which, if we as usual omit (but continue to take for granted) the condition 'P(α) > 0', is logically equivalent to (M') above:

$$(TM) \quad P(A/\alpha) > \frac{1}{2} \leftrightarrow P(\alpha \ \& \ A) > P(\alpha \ \& \ \neg A)$$

Now (TM) is not in itself a very plausible 'translation' of Hume's maxim, since, quite apart from its biconditional form, it differs from (S) and (M) in failing to provide a term-for-term correspondence with the Humean (H) – unlike 'P(A)' and 'P(A/α)', 'P(α & A)' is clearly not a permissible interpretation of 'probability(A)' (especially if we take seriously Sobel's retrospective adaptation which ensures that P(α) is always less than 1). Thus Sobel presents (TM) as his own improvement on Hume, not realizing that Hume's words (including those words which suggest a biconditional) can quite naturally be interpreted as intending the logically equivalent (M'). Confusingly, however, Sobel then goes on to provide another interpretation (this time of both halves) of Hume's maxim by suggesting that the formula

$$(4) \quad P(A/\alpha) = \frac{1}{1 + \frac{P(\alpha \ \& \ \neg A)}{P(\alpha \ \& \ A)}}$$

might be 'either what [Hume] meant to assert, or . . . something merely subtly different, if only by being more definite' (p. 236). But if Sobel is happy to find all this implicitly stated in Hume's maxim, then it is hard to see how he could possibly object to the relatively modest (M) or (M'), which, as explained above, can be straightforwardly derived from Hume's actual words.

II

The only possible defence that I can think of on Sobel's behalf is that my suggested alternative interpretation of Hume is too trivial to do justice to his thought, given that formula (M) (and indeed (M')) is a 'near tautology'. Such an objection, however, would rebound, since although (M) is admittedly virtually trivial, the same applies equally to Sobel's *logically weaker* formula (S), given the very obvious theorem

miracle against the other . . . and always reject the greater miracle'; (c) 'If the falsehood of his testimony would be more miraculous than the event which he relates; then . . . can he pretend to command my belief or opinion' (E 116).

$$P(A) \geq P(\alpha \ \& \ A)$$

which implies that (S) is a direct consequence of the ‘ \rightarrow ’ part of (TM), a formula which is itself very trivially derivable from (M’).⁶

So far as triviality is concerned, therefore, Sobel and I are in the same boat. But, perhaps unlike Sobel, I see no overwhelming objection to attributing to Hume a maxim which, when expressed in probabilistic terms, is a ‘near tautology’. And this is because I believe that Hume can plausibly here be seen, not as putting forward a new theorem to an audience already familiar with a probabilistic framework, but rather, and more fundamentally, as presenting an argument for interpreting the evidence for miracles within some such framework in the first place (though it is an open question whether Hume’s framework should be seen as *Bayesian*, given his emphasis on the ‘subtraction’ of evidential force – see for example *E* 111, 116, 127).

An example here might help to illustrate the point. Suppose that I develop a test to diagnose a debilitating genetic condition which suddenly manifests itself in middle age, but which fortunately afflicts only one person in a million. The test is fairly reliable, in that no matter who is tested, and whether they actually have the disease or not, the chance that the test will give a correct diagnosis is 99.9%, and an incorrect diagnosis only 0.1% (it is never inconclusive). A hypochondriac, Fred, who is anxious because of his approaching fortieth birthday, comes to my clinic for a test, which much to his horror (if not surprise!) proves positive. What, on the basis of this information, is the probability that Fred has the disease?

I have tried this question on various groups of students and others, who overwhelmingly conclude that Fred has good reason to be frightened, and typically estimate his chance of having the disease at 99.9%. And yet this answer is entirely mistaken, as can be seen if we imagine the effect of performing my test on the entire population of Britain (say 55 million). Of these 55 million, roughly 55 could be expected to have the disease (since it afflicts only one in a million), and it is likely that every one of these 55 will receive a positive result when tested (since the test is 99.9% probable to give a positive result for each of them individually). But now consider the 54,999,945 who do not have the disease. The vast majority of these will, of course, receive a negative result, but nevertheless 0.1% of them, or roughly 55,000, can be expected to receive an incorrect positive result. So out of 55,055 positive results overall, only 55 will be correct. Clearly a positive test does relatively little to indicate that one actually has the disease: it merely changes the relevant probability from a negligible one in a million to the only slightly more worrying one in 1001 (55 in 55,055).

That most people go badly wrong when faced with this sort of example will come as no surprise to those familiar with the famous work of Tversky and Kahneman,⁷

⁶ The reasoning in note 4 can quickly prove (M’), from which (TM) follows if the right hand side is multiplied by $P(\alpha)$ (which being greater than zero preserves the inequality). An odd feature of Sobel’s exposition is that he provides a page of ‘notes in lieu of a proof’ of (TM), when a complete formal proof could be far shorter. His proof of (S) in the appendix is also much longer and more convoluted than it need be.

⁷ See A. Tversky and D. Kahneman, ‘Evidential Impact of Base Rates’, in Kahneman, Slovic and Tversky (eds), *Judgement under Uncertainty: Heuristics and Biases*

who found that when assessing the force of some evidence α for an event A , people typically ignore $P(A)$, the initial probability of the event, and tend to estimate $P(A/\alpha)$, its probability in the light of evidence α , purely on the basis of $P(\alpha/A)$, in other words the probability that evidence α would be forthcoming if A were true. But that this is an error is surely close to the point which Hume is making in 'Of Miracles': that when assessing the evidence α for an alleged miracle A , it is not enough just to consider the usual reliability of that sort of evidence, or the fact that human testimony normally correlates well with genuine events – for when the event described would be miraculous, its immense initial improbability is almost always sufficient to dominate every other consideration, and to outweigh by far virtually any testimony that has ever been presented.

Anyone who believes that Hume's maxim, when interpreted in the way that I have suggested, is merely tautologous and hence worthless, would do well to bear in mind the example of the diagnostic test. Most people do in fact go seriously wrong when faced with this example, and yet a little reflection on Hume's admittedly 'trivial' maxim might be sufficient to protect them from error. On leaving my clinic, Fred should simply ask himself whether

the test is of such a kind, that its falsehood would be more surprising, than the disease, which it endeavours to establish.

Given that the test is wrong one time in a thousand,⁸ while the disease afflicts only one in a million, Hume's test should at least be sufficient to mitigate Fred's hypochondriac concern! Trivial it may be, but as Hume observed it can nevertheless provide a valuable 'check' to protect us from 'superstition and delusion', and from our natural human tendency to overlook completely the importance of initial probabilities when assessing the impact of evidence, testimony or otherwise, for allegedly extraordinary events.

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(Cambridge UP, 1982), pp. 153–60. Tversky and Kahneman's examples are discussed in two earlier papers on miracles, which appeared together in *The Philosophical Quarterly*, 37 (1987), by Sobel (pp. 166–86) and by David Owen (pp. 187–202).

⁸ There are hidden complications here, because as we have seen a *positive* test is wrong 1000 times out of 1001 – and analogously, at least on Hume's view, testimony for miracles is always false! This suggests that when Hume speaks of 'testimony . . . of such a kind', he is referring to the manner and circumstances of the testimony rather than to its content, and so in a parallel fashion I here ignore the outcome of the test in quoting a probability of error of one in a thousand. Much more could be said on these matters, which are related to the (generally overlooked) issue between 'type' and 'token' interpretations of Hume's maxim, but such a discussion must wait for another occasion.