

Miracles and David Hume

**Peter Millican, Hertford College, Oxford
for the Royal Institute of Philosophy, 6/2/09**

Some Basic Probability

Probabilities range from 0 to 1. If a proposition has a probability of 0, then it's certainly false; if 1, then it's certainly true. A proposition with a probability of $\frac{1}{2}$ (or 0.5, or 50%) is equally likely to be true as false, and a proposition with a probability of $\frac{3}{4}$ (or 0.75, or 75%) is three times as likely to be true as false.

If two propositions are *mutually exclusive* (i.e. they can't both be true together), then the probability that at least one of them is true is equal to the sum of the individual probabilities. So if the probability that Teasing Tom will win the race is 0.3 (or 30%), and the probability that Jumping Jane will win is 0.25 (or 25%), then the probability that *either* Teasing Tom *or* Jumping Jane will win the race is 0.55, or 55%:

$$\Pr(T \text{ or } J) = \Pr(T) + \Pr(J) = 0.3 + 0.25 = 0.55$$

If two propositions are *independent* – so the truth or falsehood of one of them has no impact whatever on the probability of the other – then the probability that *both* of them are true is simply the product of the two individual probabilities. So if Jumping Jane moves to the weaker second race to improve her chances, giving Teasing Tom a 40% chance of winning the first race, and Jumping Jane a 60% chance of winning the second, then the probability that they'll *both* win their respective races is:

$$\Pr(\text{T and J}) = \Pr(\text{T}) \times \Pr(\text{J}) = 0.4 \times 0.6 = 0.24 \text{ (i.e. 24\%)}$$

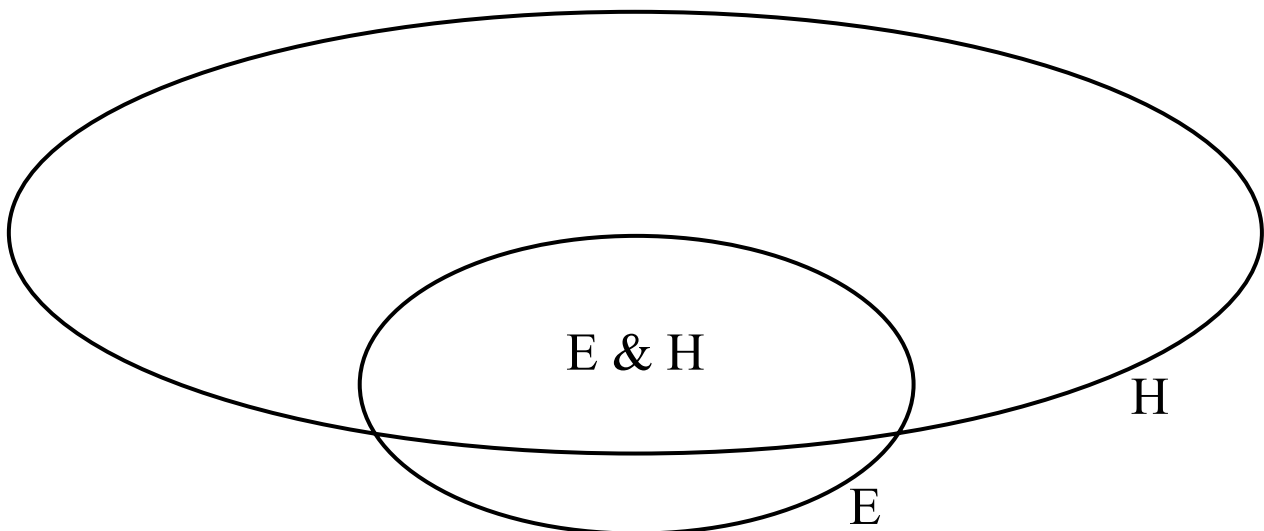
Often, the probability of one event depends on another, and we can get sequences of dependent events. Suppose, for example, I have a 0.8 probability of catching the 12:00 train for Reading (R), and *if* I manage to catch that train, I will *then* have a 0.6 probability of catching the 12:45 train for Didcot (D). The probability that I will catch both trains is then $0.8 \times 0.6 = 0.48$, slightly less than half. So the relevant equation in this case is:

$$\Pr(\text{R \& D}) = \Pr(\text{R}) \times \Pr(\text{D} \mid \text{R})$$

where “ $\Pr(\text{D} \mid \text{R})$ ” represents the probability of D *given* R.

(b) Bayes' Theorem

Suppose I hold this sheet of paper out under a greying sky, and wait until the first spot of rain drops within the area E. What is the probability that this will *also* lie within the area H?



The answer is “about 0.75”, because this is the chance that the rain drop will be in the area marked “E & H”, *given that* it is in the area E. The overlap area E & H – where *both* E *and* H are “true” – constitutes roughly three quarters of E.

It can be helpful to think of probabilities in terms of these sorts of diagrams, where the regions represent “spaces of possible situations” whose areas are proportional to their probabilities. (You might like to imagine Destiny throwing a dart randomly at the diagram to determine what is going to happen.)

Now suppose that H represents the space of possible situations in which some *hypothesis* is true, while E represents the space of possible situations in which some proposition about *evidence* bearing on the hypothesis is true.

We can calculate the probability of H given E as follows:

$$\mathbf{Pr(H|E) = Pr(E \& H) / Pr(E)}$$

“ $\text{Pr}(H|E)$ ” means *the probability of H , given that E is true* – that is, the probability to be given to H once we know that E is true (so this is called a *conditional* probability). By contrast, “ $\text{Pr}(E)$ ” is *the initial probability of E* – the probability that E had *before* we found out that it was true – and “ $\text{Pr}(E \& H)$ ” is *the initial probability of both E and H* – that is, the probability that E and H would turn out to be true, *before* we found out that E was true.

This result, a simple form of **Bayes’ Theorem**, is essential background for understanding Hume’s argument concerning miracles in probabilistic terms. In my talk, I use “ M ” (instead of “ H ”) to signify some unlikely event, and “ $t(M)$ ” (instead of “ E ”) to signify some testimony for that event. The symbol “ \neg ” is used to mean “not”.

The Diagnostic Test example

Suppose that I develop a test to diagnose a debilitating genetic condition which suddenly manifests itself in middle age, but which fortunately afflicts only one person in a million. The test is fairly reliable, in that no matter who is tested, and whether they actually have the disease or not, the chance that the test will give a correct diagnosis is 99.9%, and an incorrect diagnosis only 0.1%. Fred, a hypochondriac, anxious because of his approaching fiftieth birthday, comes to my clinic for a test, which much to his horror proves positive. On the basis of this information, is it probable that Fred has the disease?

To put his mind at ease, Fred might ask himself: “*Is the test of such a kind, that its falsehood would be more surprising, than the disease, which it indicates?*”

In fact the falsehood of the test would be much *less* surprising than it would be for Fred to have the disease. Hence if we calculate the probabilities of a true-positive and of a false-positive result, we find that the latter is far greater (by a factor of 1001, so Fred’s probability of illness is only 1 in 1002):¹

$$\text{True positive:} \quad \frac{1}{1,000,000} \times \frac{999}{1,000} = \frac{999}{1,000,000,000}$$

$$\text{False positive:} \quad \frac{999,999}{1,000,000} \times \frac{1}{1,000} = \frac{999,999}{1,000,000,000}$$

¹ Imagine the test being performed on a billion people, one thousand of whom have the disease. We’d expect 999 true positives, and 999,999 false positives (1001 times as many).

The Aleph Particle Detector example

Imagine that I am conducting an experiment on some type of sub-atomic particle – let’s call them “aleph” (\aleph) particles – created by nuclear collisions. Whenever a relevant collision takes place, various particles result, and let us suppose that 1% of these collisions will yield an \aleph particle (event “M”). My detector is highly reliable, but not infallible: if an \aleph particle is present, it will be registered with 99.9% probability, but 0.1% of those collisions that do not create an \aleph particle will also register on the detector (hence both “false negatives” and “false positives” have an identical probability of 0.1%). Now suppose that on the next collision, my detector gives a positive result (testimony “t(M)”) – should I believe it?

The initial probabilities of a positive result are:

$$\text{True positive: } \Pr(M \ \& \ t(M)) = 1\% \times 99.9\% = 0.999\%$$

$$\text{False positive: } \Pr(\neg M \ \& \ t(M)) = 99\% \times 0.1\% = 0.099\%$$

So the overall probability of a positive result = 0.999% + 0.099% = 1.098%, and applying Bayes’ Theorem:

$$\begin{aligned} \Pr(M | t(M)) &= \Pr(M \ \& \ t(M)) / \Pr(t(M)) \\ &= 0.999\% / 1.098\% = 0.9098\dots \approx 91\% \approx \frac{10}{11} \end{aligned}$$

This fits, because a True positive is around 10 times as likely as a False positive. The “testimony” of my detector is credible!

Bibliography

Earman, John (2000), *Hume's Abject Failure: The Argument Against Miracles*, Oxford University Press

Earman, John (2002), "Bayes, Hume, Price, and Miracles", *Proceedings of the British Academy* 113: 91-109

Earman attacks Hume aggressively, describing the argument concerning miracles as "tame and derivative and something of a muddle" (2002, p. 93), "a shambles from which little emerges intact, save for posturing and pompous solemnity" (2002, p. 108). However he misinterprets Hume's argument badly, and the account given in my talk is intended to refute his dismissal of it.

Garrett, Don (1997), *Cognition and Commitment in Hume's Philosophy*, Oxford University Press, chapter 7, pp. 137-62; revised version in Peter Millican (ed.), *Reading Hume on Human Understanding*, Oxford 2002: 301-32

A very useful discussion of various issues that can make Hume's argument hard to understand. Focuses on pure philosophical points rather than discussing probabilistic interpretations.

Hume, David (1748/77), *Enquiry Concerning Human Understanding*, ed. Peter Millican, Oxford "World's Classics", 2007

Gives Hume's final edition, together with other relevant Hume texts (e.g. the Abstract, "Of the Immortality of the Soul", excerpts from the Dialogues Concerning Natural Religion, and various letters). Also includes editorial material to set the work in context and assist with understanding (e.g. explanatory notes, a glossary, and a glossarial index of major philosophers and philosophical movements). The Introduction sketches the relevant history of philosophy from Aristotle to the Early Modern period, putting Hume's interests – notably religion – in a wider context.

Hume, David (1779), *Dialogues Concerning Natural Religion*, ed. Norman Kemp Smith, Nelson, second edition 1947

The funniest truly great work of Philosophy ever written! Hume's famous demolition of the Design Argument for God's existence. The Kemp Smith edition is standardly used for references, but there are several good modern editions, e.g. edited by John Gaskin in the Oxford University Press "World's Classics" series, or by Martin Bell in the "Penguin Classics" series. To start with you may find the 18th century English a bit hard to get used to, but once you're over that hurdle, it's entertaining and extremely hard-hitting.

Millican, Peter (2002), "Critical Survey of the Literature on Hume and the First Enquiry", in Peter Millican (ed.), *Reading Hume on Human Understanding*, Clarendon Press: 413-74

A survey of 250 articles and books concerning Hume's philosophy, with particular reference to the Enquiry concerning Human Understanding. Includes 5 pages on miracles, and 13+ on religion.

Owen, David (1987), "Hume versus Price on Miracles and Prior Probabilities: Testimony and the Bayesian Calculation", *Philosophical Quarterly* 37: 187-202 and reprinted in Peter Millican (ed.), *Reading Hume on Human Understanding*, Oxford 2002: 335-48

An engaging discussion of philosophical issues surrounding the probabilistic interpretation of Hume on miracles, drawing attention in particular to the work of Hume's contemporary Richard Price.